Name___________________________ **Section**_______________

Partner(s)________________________ **Date**_______________

ASTROMATH 101: BEGINNING MATHEMATICS IN ASTRONOMY

Astronomers deal with very, very large distances, some incredible temperatures, and even really, really small wavelengths. The table below gives a variety of examples showing the vast variations in the parameters listed.

Distances	Masses	Temperatures	Velocities
Earth-Sun 150 million km	electron 9.1×10^{-31} kg	surface of Sun 6000 K	light 3×10^{5} km/s
Pluto-Sun 5900 million km	hydrogen atom 1.7×10^{-27} kg	core of Sun 15 million K	asteroids 20-40 km/s
radar wavelength 10 cm	Earth 6.0×10^{24} kg	Venusian day 750 K	Earth in orbit 30 km/s
x-ray wavelength 1×10^{-9} m	Sun 2.0×10^{30} kg	Earth day 300 K	Earth's rotation at equator 1660 km/hr

Many relationships in astronomy are presented as equations or in graphical format. Hence, to understand many of the concepts in this course, you need some mathematical savvy. Let's look at some basics. A TI-83 (or 82) graphing calculator, which is required in MAT classes, would be helpful to have for this activity. Key strokes are shown in bold with [ ], such as **[ON]**.

$$1{,}000{,}000 = 10^6 = \text{million}$$
$$1{,}000{,}000{,}000 = 10^9 = \text{billion}$$

A fundamental unit in astronomy is the speed of light which is 299,792.5 kilometers per second (km/s). You may travel on the beltway at 55 miles per hour (mph) or about 88 km/hr. If you are going 55 mph, how far would you travel in 2.5 hours?

Did you get about 138 miles! Now how long would it take to drive 275 miles at 55 mph?

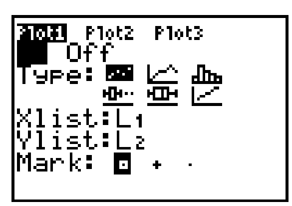
It would take you 5 hours. Now let's deal with the speed of light which we will express as 3.0 x 10^5 km/s. Use the **[2nd] [EE]** to enter numbers in scientific notation. The calculator shows an "E" in place of the "10." How long does it take the Sun's light to reach the Earth, which is 150 million kilometers from the Sun in seconds?   In minutes?

It actually takes light 8.3 minutes to travel from the surface of the Sun to the Earth.

Two students go for a ride in their car at 2 am on the beltway. The driver keeps the car at 60 mph as the second person records the mileage traveled and time. Plot a graph of the data given below using graph paper. Since distance or mileage traveled depends on time, the distance is plotted on the y-axis and time on the x-axis. Now let's use the TI-83 graphing calculator. Key strokes are shown with [ ] in bold, such as **[ON]**. Using **[STAT]** EDIT **[ENTER]**, this puts you into the lists (spreadsheet).   Enter time in L_1 and distance in L_2.

Time, hr	Distance, miles
0.5	29
1.0	61
1.5	89
2.0	122
3.0	178
5.0	306

Plot the data using **[2nd] [STAT PLOT]** and select PLOT1. Set up as shown on the screen. Press **[ZOOM] [9]** to see graph.



What type of relationship do you find?   Linear or non-linear

On your hand-drawn graph, draw the best-fit straight line through the data points. Calculate the rate of change or slope of the line using two points (x_1,y_1 and x_2,y_2) on the line (NOT DATA POINTS).

$$SLOPE = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$

What are the units for the slope in this problem?

The slope is the **velocity** in this problem which should come out to 60 miles/hour.

What would the line look like for the same trip at 40 miles/hour? Sketch a dashed line (---) on the graph. How far would you have traveled in 5.0 hours? Explain how you determined this.

Let's take a classical example of historical data to demonstrate modeling. The table below lists the nine planets in our solar system along with their distances from the Sun, in AU (relative to Earth at 1 AU), and periods, in Earth years, time to revolve once around the Sun.

Planet	Distance, d (AU)	Period, P (years)
Mercury	0.3871	0.24084
Venus	0.7233	0.61515
Earth	1.0	1.0
Mars	1.5237	1.8808
Jupiter	5.2028	11.862
Saturn	9.5388	29.456
Uranus	19.191	84.07
Neptune	30.061	164.81
Pluto	39.529	248.53

Is there a relationship between distance and period? Plot a graph of period on the y-axis against distance on the x-axis using the TI-83 instructions on the previous page. Is the plot linear?

Sketch the graph and label the axes.

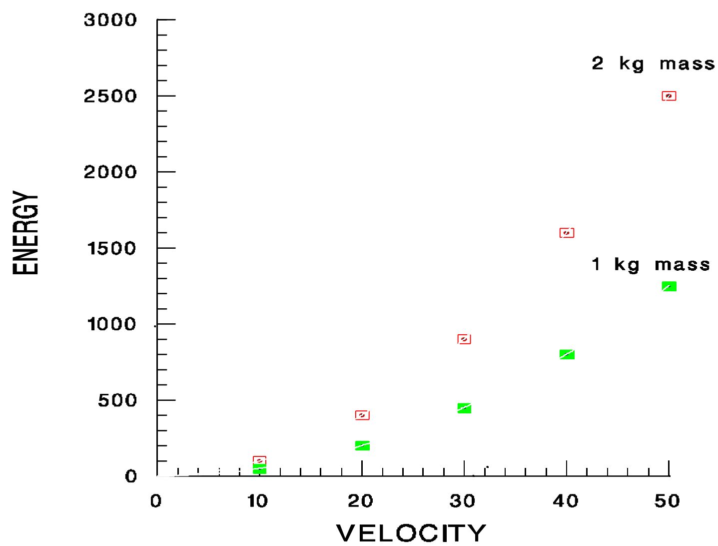
What type of relationship do you think exists?

Press **[Y=]** to get into the function editor. In Y_1: type X^1.5 using the **[X,T,è,n]** to get the X, and then press **[GRAPH]**. You are plotting the function $Y = X^{1.5}$. How well does it fit?

This is Kepler's third law of planetary motion discovered in 1619. In most astronomy books the standard form given is by $P^2 = d^3$ (the function above squared), where P is in Earth years and d in AU's. If x = distance (d) and y = period (P), you can rewrite $y = x^{1.5}$ to be $P = d^{1.5}$, which is the same as $P^3 = d^2$. Kepler only had five planets to work with and no electronic devices to help with calculations.

Like Kepler, we now have discovered a mathematical model or an equation that relates period and distance. Next we want to test our model. How good is it? We want to make predictions and if possible verify them. In 1687, Newton verified Kepler's third law using the moons of Jupiter and then Saturn. The asteroid belt is located between the orbits of Mars and Jupiter at a typical distance of 3 AU. What should be the period of these asteroids? Use the [TRACE] key and then the cursor up, **[>]** to be able to trace along the function using the **[<]** or **[=]**. You could solve the equation too!

The kinetic energy (E) of an object such as an asteroid is dependent on the mass (m) and velocity (v) of the object. Consider the graph below for two different mass objects.



For any velocity, how does the energy change when the mass doubles?

For a 2 kg mass, if you double the velocity, what happens to the energy?

Sketch the curve for an object with a mass of 3 kg as a dashed line (---) on the graph above.

Since E = (½)mv^2, when the mass doubles so does the energy and when the velocity doubles the energy quadruples.

Practice Problems

1. Electromagnetic radiation in the visible range has wavelengths of 400 to 700 nanometers, nm. A nanometer is a billionth of a meter, 1 nm = 10^{-9} m. What is the visible range in meters?

2. An asteroid, about the size of a softball, is traveling at 40 km/s. What is its velocity in miles per hour, mph? Should you try to catch it? Explain why or why not.

3. The astronomical unit, AU, is the distance between the Sun and Earth, 1 AU = 150 million miles. If Venus is 0.72 AU from the Sun and Saturn is 9.5 AU, what are their distances in kilometers?

4. The universal law of gravitational attraction is given by

$$F = \frac{Gm_1m_2}{r^2}$$

where F is the force of gravitational attraction, m_1 and m_2 the masses of two objects, r the distance between the objects, and G is a constant.

Using your TI-83, let's see how the force of gravitational attraction, F, is influenced by first changing the mass of one of the objects, m_2 and then changing the distance, r, between objects. For the equations below, the left side shows the two situations described above with the variables that are constant in parentheses. The equations to the

$$F = (Gm_1m_2)\frac{1}{r^2}$$

right with x and y are the same equations, simplified to enter into the calculator.

case I: changing the mass

case II: changing the distance between objects

$$F = \left(\frac{Gm_1}{r^2}\right) m_2$$

$$y_2 = (k)\frac{1}{r^2} \ or \ \frac{(k)}{r^2}$$

$$y_1 = (k)x$$

Using the function editor, **[Y=]**, enter the two equations. For simplicity we will assume k = 1 in both cases.

Y_1= X
Y_2= 1/X^2

Using **[WINDOW]** set the following:

Xmin = 0	Xmax = 4	Xscl = 1
Ymin = 0	Ymax = 4	Yscl = 1

Now press **[GRAPH]**, the graphs are plotted in sequence Y_1 then Y_2. (You can plot them one at a time by removing the highlight on the = in the function editor.)

Sketch the two plots below and label the axes:

Describe what the graphs show for the two variables involved.

5. Look up the average densities in your textbook for the nine planets. Using the distances from the Sun in AU for each planet given earlier, plot density (y-axis) against distance (x-axis). Sketch it below.

Describe the graph. Mercury, Venus, Earth, and Mars are called the inner or terrestrial planets, while Jupiter, Saturn, Uranus, Neptune, and Pluto are the outer planets. Jupiter, Saturn, Uranus, and Neptune are known as the jovian or gas giants. Explain this based on density.